5. That occasionally, though rarely, the summation of rapidly succeeding nervous impulses may be only incompletely effected within the nerve-cells of the spinal cord, or may not occur at all. In these cases results similar to those of Franck and Pitres are obtained.

A more detailed account of these experiments will shortly be published in the "Journal of Physiology."

December 17, 1885.

Professor G. G. STOKES, D.C.L., President, in the Chair.

The Presents received were laid on the table, and thanks ordered for them.

Professor Horace Lamb (elected 1884) was admitted into the Society.

The following Papers were read :—

I. "An Experimental Investigation into the Form of the Wave Surface of Quartz." By JAMES C. MCCONNEL, B.A. Communicated by R. T. GLAZEBROOK, M.A., F.R.S. Received November 9, 1885.

(Abstract.)

The paper contains an account of a number of measurements of the well-known "dark rings" of quartz. Each ring is due to one wave being retarded in the quartz behind the other by an integral number of wave-lengths, so the measurements give the directions through the plate of quartz corresponding to a series of known retardations. The relative retardation is, especially in a crystal of weak double-refracting power like quartz, mainly dependent on the distance between the two sheets of the wave surface. Thus my observations really give the separation between the two sheets at various points, and it is in this separation that the peculiarities of quartz are most strongly marked, and the various expressions put forward by theory most widely divergent.

With a plate cut at right angles to the axis, I obtained values of the separation from $\phi=4°$ to $\phi=39°$—ϕ being the angle between the ordinary wave normal and the axis—and with a plate cut parallel to the axis I obtained values from $\phi=53°$ to $\phi=90°$.

An obvious danger in this mode of investigating the wave surface

is the influence of the refraction effects. This is discussed at some length in the paper, and is shown to be in all probability negligible.

The measurements were taken with a spectrometer fitted up as a polariscope, whose great focal length and finely graduated circle were of much service.

I found it convenient to treat separately the region near the axis, where the abnormal form of the wave surface of quartz is most obvious. I have compared my results with nine different theories, each of which gives an expression of one of the two following forms.

$$D^2 = P_1^2 \sin^4 \phi + D_0^2.$$
$$D^2 = P_2^2 \sin^4 \phi + D_0^2 \cos^4 \phi.$$

Here D is "the number of wave-lengths by which one wave lags behind the other in air, after the light has traversed normally a plate of quartz one millimetre thick, the normal to whose faces makes an angle ϕ with the optic axis." D_0 is the value of D when $\phi=0$, and is known from the rotatory power, and P_1 and P_2 are constants to which the theories assign different values. By inserting the observed values of D and ϕ, I obtained values of P_1 and P_2 from each ring. The results from one plate about 20 mm. thick were as follows :—

ϕ	4° 24′	5° 51½′	6° 51½′	7° 40½′	8° 23½′	9° 38′	11° 41′
P_1	15·054	15·207	15·220	15·260	15·249	15·258	15·269
P_2	15·220	15·293	15·290	15·311	15·295	15·292	15·292

Similar results were obtained from a second plate about 27 mm. thick.

From these figures I concluded that the second expression was the correct one, and that $P_2 = 15{\cdot}30 \pm {\cdot}01$. There is a considerable discrepancy in the case of the first ring, of which two possible explanations are given in the paper.

Cauchy gives $P_2 = \frac{a-b}{a^2\lambda} = 15{\cdot}351$, where a and b are the wave velocities perpendicular to the optic axis.

$$\text{Lommel gives } P_2 = \frac{1-a^2}{1-b^2}\,\frac{a+b}{2a}\,\frac{a-b}{a^2\lambda} = 15{\cdot}178.$$

$$\text{Kettler} \quad ,, \quad P_2 = \frac{a+b}{2b}\,\frac{a-b}{a^2\lambda} = 15{\cdot}486.$$

$$\text{Sarrau} \quad ,, \quad P_2 = \frac{a+b}{2a}\,\frac{a-b}{a^2\lambda} = 15{\cdot}306.$$

The other five, MacCullagh, Clebsch, Lang, Boussinesq, and Voigt, have the first form of expression and give $P_1 = \frac{a+b}{2a}\,\frac{a-b}{a^2-\lambda} = 15{\cdot}306.$

Thus Sarrau alone succeeds in explaining the observations satisfactorily.

For the larger values of ϕ, I calculated on Sarrau's theory what value of $a-b$ was required to give the retardation observed in each ring, and obtained as follows:—

ϕ	15° 14′	18° 21′	23° 16½′	28° 7½′	32° 7′	35° 34′	38° 36′
$a-b$ ...	·0037916	·0037922	·0037927	·0037931	·0037939	·0037932	·0037936

The observations on the plate cut parallel to the axis gave—

ϕ	53° 58′	57° 11′	64° 45′	72° 13′	79° 53′	83° 40′	85° 37′
$a-b$ ...	·0037949	·0037947	·0037945	·0037944	·0037943	·0037946	·0037944

In this last set the retardation amounted to about 300 wave-lengths, so that the bands corresponding to the two D lines were considerably separated, and the bands seen were consequently ill defined. Fortunately, however, a large angular error only produced a small change in $a-b$, so that these results are far more accurate than those of the other two sets. For $\phi > 50°$ the rotatory term is negligible, and Sarrau's theory is reduced to the simple Huyghenian construction. So our observations show that for $\phi > 50°$ the Huyghenian construction represents the wave surface in quartz with great accuracy.

From $\phi=15°$ to $\phi=50°$ there is a slight but persistent increase of $a-b$ with ϕ. It is shown in the paper that this may be satisfactorily accounted for by using the more general form of Sarrau's wave surface, viz., $(s^2-a^2)(s^2-a^2\cos^2\phi-b^2\sin^2\phi)$.

$$=\frac{4\pi^2a^2}{\lambda^2}(g_2\cos^2\phi+f_1\sin^2\phi)(g_2\cos^2\phi-g_1\sin^2\phi).$$

Here s is the wave velocity and a, b, g_2, f_1, g_1 are constants, g_2 depending on the rotatory power. Up to this point we have been supposing f_1 and g_1 are negligible. If now we make $8\pi^2a^2g_2(g_1-f_1) = {·}00033(a^2-b^2)^2\lambda^2$, we find that Sarrau's theory agrees with observation throughout within the limits of experimental error, even in the case of the first ring.

The observations were taken in the Cavendish Laboratory, Cambridge, during the months of March and June, 1885.

For full details as to the apparatus, the plates of quartz used, the mode of observation, the precautions necessary, the temperature effects, and the calculations, reference must be made to the paper.